**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: B

(1-pnorm(50,45,8)) = 0.265985529048701

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS: False

Mean=38 , SD = 6

Z Score for 44 = (44-38)/6

=6/6 = 1 = 84.13%

People above 44 age =100-84.13

= 15.87% = 63 out of 400

Z Score for 38 = (38-38)/6 = 0 = 50%

Hence people between 38 and 44 = 84.13 – 50 = 34.13%

Therefore more employees at the processing center are older than 44 than between 38 and 44 is **False**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS: True

Z=(X-µ)/ *σ*

P(X≤30)=p(Z≤(30-38)/6)=p(Z≤-1.33)= 0.0918(using z table)

Expected count=0.0918\*400= 36.72

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: 2 *X*1  will be greater scale version than *X*1 + *X*2 . If *X*1 and *X*2 are normally distributed then the sum of the random sample will be exactly same

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

ANS: D

qnorm(0.995,100,20)

qnorm(0.005,100,20)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Given that:

$1 = Rs. 45

Profit 1∼N(5,3 ^2)

Profit 2∼N(7,4 ^2)

Thus,

Company's profit:

P = N( 5+7, 3^2 + 4^2) = N(12, 5^2)

**A**):

95% of the probability lies between 1.96 standard deviations of the mean.

Thus range is:

= (12 - 1.96 \* 5, 12 + 1.96 \* 5)

= ($2.2M, $22.8M)

= (Rs.99M, Rs.1026M)

**B**): Fifth percentile is calculated as:

P(Z <={p-12}/{5}) = 0.05

From p values of z score table, we get:

{p-12}/{5} = -1.644

p = 12 - 8.22 = 3.78

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.

Or 5th percentile of profit is Rs. 170.1 Million.

**C**): Loss is when profit < 0

Thus: p < 0

The first division of company, thus have larger probability of making a loss in a given year.